ပြည်ထောင်စုသမ္မတမြန်မာနိုင်ငံတော်အစိုးရ

ပညာရေးဝန်ကြီးဌာန

အဆင့်မြင့်ပညာဦးစီးဌာန

ကျိုင်းတုံတက္ကသိုလ်

(၃)

ဂုဏ်ပြုမှတ်တမ်းလွှာ

၂၀၁၈ ခုနှစ်၊ ကျိုင်းတုံတက္ကသိုလ် (၁၁)နှစ်မြောက် နှစ်ပတ်လည်နေ့အထိမ်းအမှတ် သုတေသနစာတမ်းဖတ်ပွဲတွင် ဖတ်ကြားခဲ့သည့် စာတမ်းရှင်အမည် ဒေါက်တာ [illegible] ရာထူး [illegible] ၊ ဌာန သင်္ချာ အား ဤဂုဏ်ပြုမှတ်တမ်းလွှာကို ချီးမြှင့်လိုက်သည်။

ဒေါက်တာ စမ်းစမ်းမာ

ပါမောက္ခချုပ်

ကျိုင်းတုံတက္ကသိုလ်

ရက်စွဲ၊ ၂၀၁၈ ခုနှစ်၊ ဇန်နဝါရီလ (၂၂)ရက်

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Content

Computing Heat Equation Using Matlab

Thet Mon Win*

Abstract

In this paper, the heat equation problems are discussed. And then, some theorems, which provided Fourier's series and Finite Element Methods are expressed. We used Matlab's solver to model the heat diffusion and showed graphical solutions for some problems.

Introduction

Partial Differential Equations occur in various physical and engineering problems when the functions involved depend on two or more independent variables. An equation involving one or more partial derivatives of an unknown function of two or more independent variables is called a partial differential equation. The most general second-order linear partial differential equation in two independent variables is

$$a\frac{\partial^2 z}{\partial x^2}+b\frac{\partial^2 z}{\partial x\partial y}+c\frac{\partial^2 z}{\partial y^2}+d\frac{\partial z}{\partial x}+e\frac{\partial z}{\partial y}+fz=g \qquad (1)$$

where a, b, c, d, e, f and g are functions of x and y. If $g = 0$, Equation (1) is said to be homogeneous, otherwise, it is non-homogeneous.

Heat Equation

One-dimensional Heat Flow

The flow of heat in a long homogeneous bar (or wire) of length ℓ and cross-section α sq.cm. Let $\rho(\text{gm/cm}^3)$ be the density, s the specific heat (in cal/gm deg) and k the thermal conductivity of the material. Let one end of the bar be taken as origin, the direction of heat flow along the bar as the X-axis. Assume that the sides of the bar are so insulated that heat flows in the direction of OX only. Let u(x, t) be the temperature at a distance x at time t. It is known that heat flows in the direction of decreasing temperature and also that the rate of heat flow is proportional to the gradient temperature $\frac{\partial u}{\partial x}$. Let PQ be an element of the bar where P = (x, y) and $Q = (x+\Delta x, y)$. The rate of heat flow across an area is proportional to the area and also to the temperature gradient normal to the area. The constant of proportionality is called the

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…h is called the one-dimensional heat equation. The constant a^2 is called diffusivity.

To solve equation (3), apply the method of separation of variables. Let the solution of (3) … the form

$$u = XT$$

…ere X is a function of x only and T is a function of t only. Then

$$\frac{\partial^2 u}{\partial x^2} = X''T \quad \text{and} \quad \frac{\partial u}{\partial t} = XT'.$$

…ubstituting these derivatives in equation (3),

$$XT' = a^2 X''T$$

$$\frac{T'}{T} = a^2 \frac{X''}{X}.$$

…nce each ratio must be a constant ,

$$\frac{T'}{T} = a^2 \frac{X''}{X} = -p^2 a^2$$

$$\frac{1}{T}\frac{dT}{dt} = -p^2 a^2$$

…ntegrating both sides

$$T = e^{-p^2 a^2 t}$$

$$a^2 \frac{X''}{X} = -p^2 a^2$$

$$\frac{X''}{X} = -p^2$$

$$X'' + p^2 X = 0$$

$$\lambda^2 + p^2 = 0$$

$$\lambda = \pm pi$$

$$X = C_1 \cos px + C_2 \sin px$$

ubject to the conditions

$$u(0,t) = u(1,t) = 0, \quad t > 0$$

$$u(x,0) = 0, \quad 0 < x < 100$$

$$u(0,t) = 0, \quad t > 0$$

$$u(100,t) = 0, \quad t > 0.$$

The constant A has value 0.208 cm^2/s.

```
>> A=0.208;
syms n x pi
100*int(10^(-7)*sin(n*pi*x/100),x,0,100)
c=ans
syms t s
int(exp(-A*n^2*pi^2*(t-s)/100^2)*c,s,0,t)
a=simplify(ans)
s=symsum(a*sin(n*pi*x/100),n,1,10);
s600=subs(s,t,600);
xx=linspace(0,100,201);
plot(xx,subs(s600,x,xx))
ans = -(cos(pi*n) - 1)/(5000000*pi*n)
c = -(cos(pi*n) - 1)/(5000000*pi*n)
ans = (exp(-(13*pi^2*n^2*t)/625000)*(cos(pi*n) - 1))/(104*pi^3*n^3) - (cos(pi*n) - 1)/(104*pi^3*n^3)
a = (exp(-(13*pi^2*n^2*t)/625000)*(cos(pi*n) - 1))/(104*pi^3*n^3) - (cos(pi*n) - 1)/(104*pi^3*n^3)
```

It appears that 20 terms is enough for a qualitatively correct graph at t=600.
First, we create the two one-dimensional grids:

```
x=linspace(-5,5,41)';
y=linspace(0,10,41)';
>> [X,Y]=meshgrid(x,y);
>> z=1./(1+(X+0.5*Y).^2);
surf(x,y,z);view(20,45)
```

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